An Investigation of Preservice Teachers' Understanding of the Area of a Parellelogram

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I hear and I forget. I see and I remember. I do and I understand.
—Chinese Proverb

Question: "How do you know that the area of a rectangle is length times width?"

Preservice Teacher 1: "From ages past, I guess. It's just the way it was always taught to me . . . something we learned back in junior high or late elementary school."

Preservice Teacher 2: "I memorized it . . . as a formula that they gave us."

Responses like these probably do not come as a shock, as many researchers after analyzing the video component of the Third International Mathematics and Science Study (TIMSS) have documented that

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U.S. students spend most of their time in the mathematics classroom practicing routine procedures as opposed to inventing, analyzing, and proving them (Stigler & Hiebert, 1997; Stigler, Gonzalez, Kawanaka, Knoll & Serrano, 1997; Stevenson & Nerison-Low, 1998). What is most unsettling about responses like these is that they were given by two preservice K-8 teachers who, undoubtedly, will be faced with the challenge of teaching measurement topics, such as area and perimeter, to young students. Although both of these preservice teachers were able to immediately state the formula for the area of a rectangle when asked, neither could explain why the formula worked, only that "It was drilled into me."

The challenge for these preservice teachers, and many other preservice and inservice teachers, lies in creating a classroom environment where both procedural and conceptual understanding on the part of young students is encouraged and modeled so that the "whys" are no longer ignored and left unanswered. This is imperative, especially when teaching the topic of measurement, as measurement is "an intrinsic part of our everyday living…Middle grades students encounter measurement ideas both in and out of school, and measurement experiences show them practical applications of mathematics" (NCTM, 1994, p. 1).

The Study

In the winter of 2001, three preservice K-8 teachers were videotaped as they solved two sample measurement problems that were also completed by students in grades 7-8 (population 2) in the TIMSS study (Adams & Gonzales, 1996). The three participants, all of whom volunteered for this study, were university-level students pursuing K-8 teaching certification. For both problems, the international average percent of TIMSS students responding correctly was 40 percent. The first problem (see Figure 1) served as a "warm-up" problem, chosen to prompt the participants to begin thinking about the concept of area.

The second problem (see Figure 2) was of more interest to the

Figure 1
First TIMSS Measurement Problem
Completed by Participants in Study

The length of a rectangle is 6 cm, and its perimeter is 16 cm. What is the area of the rectangle in square centimeters?

researchers in terms of what problem-solving strategies the preservice teachers might use to solve this problem. Throughout the solving of the two problems, the participants were constantly prompted to fully explain their thinking.

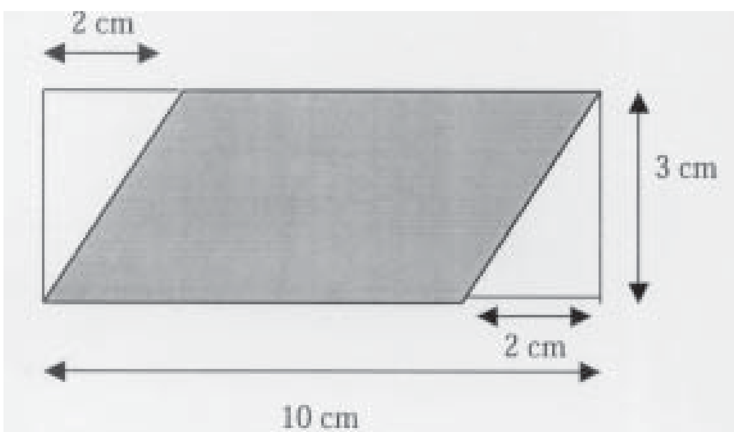
Results

All three students completed both problems correctly and in a very similar fashion; however, the second problem did pose more of a challenge to the participants since none of the three students recalled the existence of a formula for the area of a parallelogram. Thus, they were challenged to employ some other problem-solving strategies.

Upon seeing the second problem, one participant's immediate reaction was, "Uh oh. Oh my. This is what got me on the [math] entrance exam." Consequently, she, like the other participants, hesitated quite frequently during the completion of this problem, unsure of how to attack it, since she described this shape as "unusual"; that is, "uncommon." Additionally, the participants appeared less confident in their solutions, in contrast to the first problem, since they lacked the security of having a formula to draw upon. In fact, at the end of the interview, one student

Figure 2
Second TIMSS Measurement Problem
Completed by Participants in Study

The figure shows a shaded parallelogram inside a rectangle.



What is the area of the parallelogram?

commented, "You'll have to tell me if I'm right or not or I am going to be curious for the rest of the day!"

In solving this problem, all three participants began by first finding the area of the rectangle and then subtracting the area of the two triangles from this amount (see Figure 3 for a sample of one participant's work) in order to obtain the area of the parallelogram.

This is a very logical, although more time-consuming and labor intensive, solution to this problem, especially for any student who does not know, recall, or choose to use the area formula for a parallelogram. Surprising to the researchers was the fact that the participants were unable to recall the relatively simple area formula for a parallelogram, despite being able to instantly provide the very similar area formulas for rectangles and triangles. When asked whether a formula existed for the area of a parallelogram, the participants commented:

> There might be, but this was easier for me to remember when I was doing it. So the less formulas to remember, the easier it is.
> I think there is, but this one works just as well.
>
> If there is, I don't remember it . . . I would guess that perhaps there's not . . . since it's more of an unusual shape.

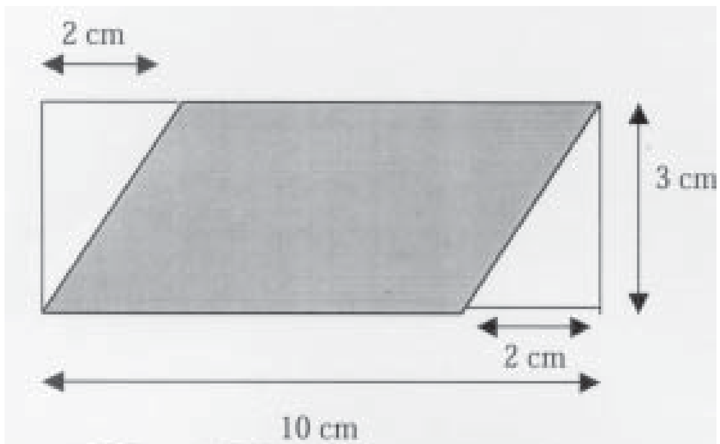
More importantly, despite the fact that all three participants exuded 100 percent confidence in stating and using the area formulas for rectangles and triangles when solving this problem, they were unable to explain or model *why* these formulas worked. When reminded that they would be teaching this very topic to young students, despite possessing a deeper understanding of the concepts, one preservice teacher admitted, "Well, gosh, I guess I'm not prepared to teach it yet!" Another acknowledged the importance of being able to articulate the "whys" because "Every kid is going to ask, 'Why do we have to know this formula?' So you have to have a reasonable sense of why you have that particular formula and where it came from." Several studies (Hiebert & Wearne, 1985, 1986; Wearne & Hiebert, 1989) have demonstrated how students, when faced with computational tasks, frequently apply rules that they do not understand. Similarly, these preservice teachers were also applying rules, in particular, area formulas, that they admitted to not conceptually understanding.

Implications and Recommendations

The NCTM (2000) acknowledges that measurement "lends itself especially well to the use of concrete materials"(p. 44) and emphasizes that students "develop formulas and procedures meaningfully through

Figure 3
Sample of One Participant's Work—Solution to Second TIMSS Measurement Problem Completed by Participants in Study

The figure shows a shaded parallelogram inside a rectangle.



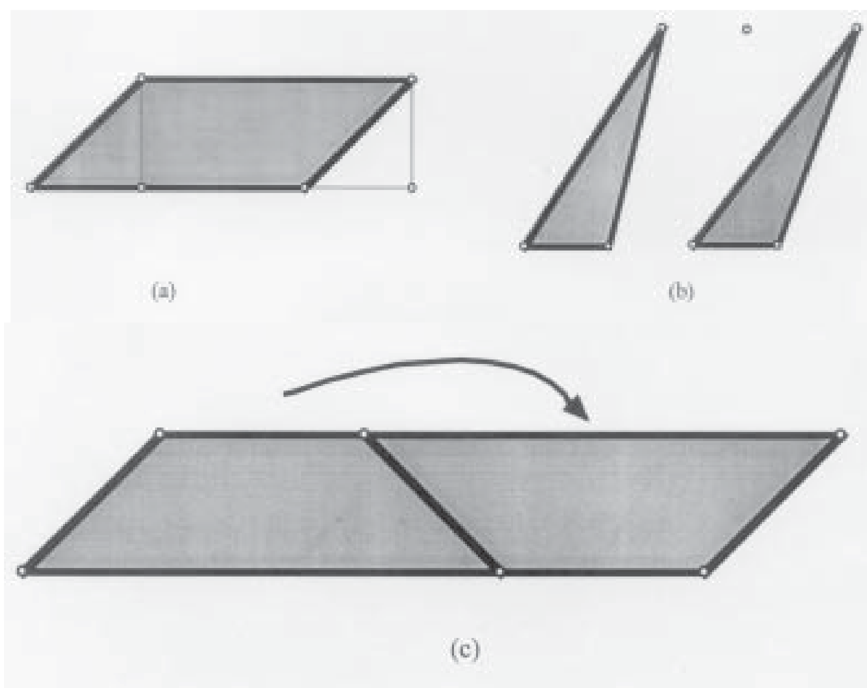
What is the area of the parallelogram?

$$A_r = 10cm \times 3cm \qquad A_T = \frac{1}{2}bh$$
$$A_r = 30cm^2 \qquad = \frac{1}{2}(2cm \times 3cm)$$
$$= \frac{1}{2}(6cm^2)$$
$$2(A_T = 3cm^2)$$
$$A_r - 2(A_T) = A_p$$
$$30cm^2 - 2(3cm^2) = 24cm^2$$

investigation rather than memorize them" (p. 244). Moreover, the NCTM warns of the "underlying confusions" that may interfere with their working meaningfully with measurement if students were to "move rapidly to using formulas without an adequate conceptual foundation in area and volume" (p. 242). Four activities that illustrate the area formula for a parallelogram and which embrace the spirit of the NCTM are: (1) decomposing a parallelogram into a rectangle, (2) decomposing a parallelogram into two triangles, (3) manipulating two trapezoids to form a parallelogram, and (4) showing the relationship between rectangles and parallelograms using technology such as Geometer's Sketchpad.

In a paper cutting activity, students can decompose a parallelogram in such a way to create one rectangle or two triangles (see Figure 4). By rearranging the parallelogram's component parts without overlapping, students could use their knowledge about the area formulas for rectangles and triangles to deduce the area of a parallelogram (NCTM, 2000, p. 244). For example, when a parallelogram is cut into two equal triangles, students can see that the area of a parallelogram is nothing more than two times the area of a triangle. Given that the area of a triangle is one-half its base times its height, doubling this yields the area formula, base times height, for a parallelogram. In a similar activity (Dolan, Williamson, and Muri, 1997, p. 250), students can discover the

Figure 4
Students Can Use Their Knowledge of the Area Formulas for Rectangles (a), Triangles (b), and Trapezoids (s) To Deduce the Area Formula of a Parallelogram



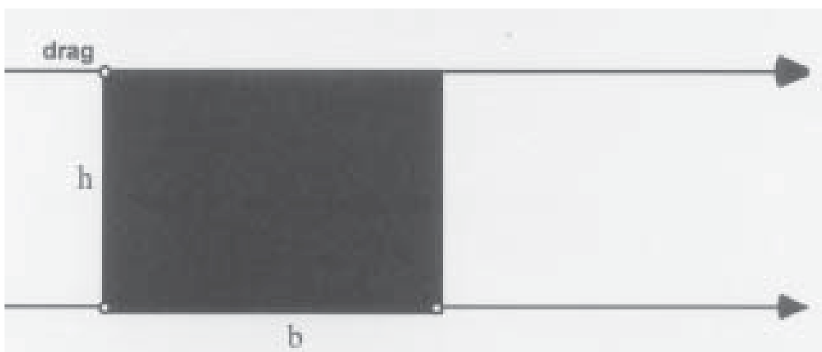
connection between parallelograms and trapezoids by manipulating two trapezoids to form a parallelogram, enabling students to better understand these shapes' area formulas as well (see Figure 4).

Using Geometer's Sketchpad (Jackiw, 1993), students can dynamically explore and make predictions about the area of a parallelogram. For example, by clicking on the top left vertex of the rectangle and dragging it to the right (see Figure 5), students can manipulate the rectangle to form a parallelogram and thus begin to observe, discuss, and explore the relationship between the area of the two shapes.

Figure 5
Geometer's Sketchpad Example

Parellelogram Area
D. Bennett 3/91

What's the area of the rectangle in terms of b and h? Drag to create a parallelogramn. How does its area compare to the area of the rectangle?



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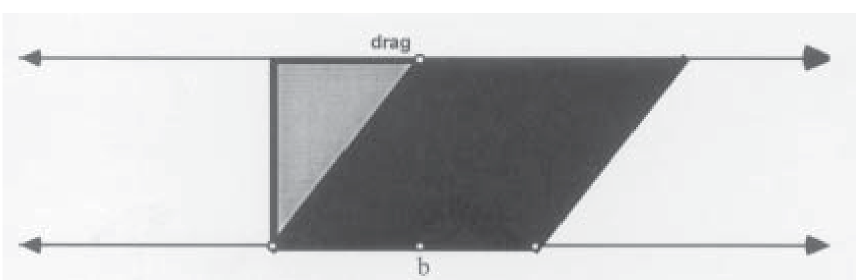

Two of the four activities were carried out with the participants in this study following the initial interview in an attempt to provide the participants with a "meaningful investigation" of the area formula for a parallelogram. After decomposing a paper parallelogram into a rectangle, one preservice teacher, excited by the prospect that she had possibly deduced the area formula for a parallelogram, proudly and confidently announced that the area formula must be length times width noticing that "It's just like this rectangle." Upon clarifying that the normal convention is to use the terms base and height for length and width, respectively, this preservice teacher's spirits were a bit deflated as she commented with dissatisfaction, "I wonder why they didn't just change the terminology for a rectangle to be the same as what they use for a parallelogram and then it would be the exact same formula. Wouldn't that have been easier to teach to children? One formula and it works for both?" This preservice teacher's comment brings to the forefront the importance of mathematical dialogue in which the distinction between and among the terms length, width, base, and height are visually and verbally made clear to students.

A different preservice teacher, after decomposing a paper parallelogram into two triangles, quickly realized the formula for the area of a parallelogram stating, "Oh, I see! It's going to be base times height because it's two triangles. And a triangle is 1/2 base times height, and if you put two together, it's just base times height…one-half plus one-half is two halves which is a whole." Because of the similarity in terminology, this investigation might provide students with a clearer understanding of the reasoning behind the area formula for a parallelogram.

Closing

The enthusiastic and surprised reaction of these preservice teachers upon learning that they had correctly deduced the area formula for a parallelogram through their own independent investigation reinforced how simple and vital it is to engage students in the exploration and discovery process of formulas as a means of enhancing their understanding of area, not to mention other measurement topics. Based on the findings of the TIMSS video study, Stigler and Hiebert (1997) claim that "U.S. teachers are still emphasizing the acquisition and application of skills" (p. 19) in contrast to their Japanese counterparts who spend more time "inventing, analyzing, and proving." This appeared to be true of the participants in this study.

Given the "practicality and pervasiveness of measurement in so many aspects of everyday life" (NCTM, 2000, p. 44), the teaching of measurement would be best approached through the frequent use of hands-on explorations and the integration of technology, as exemplified in the four activities described in this article. "Learning mathematics with understanding is essential" (NCTM, 2000, p. 20), and what is even more essential is to ensure that our preservice teachers possess this understanding and instill it in their students. As the Chinese proverb reminds us, by engaging in the "doing," students will indeed understand.

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