Using the Performance Assessment for California Teachers to Examine Pre-Service Teachers' Conceptions of Teaching Mathematics for Understanding

Elizabeth A. van Es
& Judi Conroy
University of California, Irvine

Motivated by a concern about the low performance of American students in mathematics, the National Research Council published the report *Adding It Up: Helping Children Learn Mathematics* (2001). This report summarizes a core body of research revealing that U.S. students continue to perform poorly in mathematics. While they can carry out straightforward procedures, American students demonstrate limited understanding of mathematical concepts and are unable to apply their knowledge to solve novel problems. This report also explains that the preparation of elementary and middle school teachers falls short of equipping future teachers with the knowledge they need to help students develop mathematical proficiency. Thus, a key challenge for mathematics teacher education is to prepare prospective teachers to teach mathematics for understanding. As Philipp (2008) describes, meeting this challenge is difficult because "teachers lack the depth and flexibility of mathematical understanding and the corresponding beliefs they need to teach for proficiency (NRC, 2001)" (p. 3). Moreover, given the persistence of traditional teaching practices in American teaching, prospective teachers have had few, if any, opportunities to participate in classrooms that promote learning mathematics for understanding (Lortie, 1975; Zeichner & Liston, 1987).

Elizabeth A. van Es is an assistant professor and Judi Conroy is the director of student affairs, both with the Department of Education at the University of California, Irvine. Irvine, California.

Teacher education programs have adopted several approaches to help future teachers develop the knowledge, skills, and practices for teaching mathematics for understanding. Some examples include using narrative and video cases that illustrate this model of mathematics teaching and engaging future teachers in the analysis of these cases (Hatfield & Bitter, 1995; Lampert & Ball, 1998; Santagata, Zannoni, & Stigler, 2007) and designing courses that integrate the development of pedagogical content knowledge through the examination of mathematics and mathematics pedagogy (Philipp, 2008). Still, we know very little about the particular ways that pre-service teachers have come to understand teaching mathematics for understanding, as well as what practices they perceive will help them accomplish this goal. As Boaler and Humphreys (2005) describe, an important direction of teacher research involves understanding teachers' decision making in the moment of teaching. They write:

> Each [pedagogical] move is important and demonstrates the complexity of teachers' work. Such moves also demonstrate the level, or "grain size," at which teaching decisions are made. Teachers are often offered advice that is at a much bigger grain size, such as whether to use group work to have discussions or lecture. We see […] that teachers need to make decisions that are at a smaller grain size, such as when and how to curtail a discussion, which examples of representation to use, or which students to call upon. The field of educational research has not developed extensive knowledge of the detailed pedagogical practices that are helpful for teachers to learn, yet the difference between effective and ineffective teaching probably rests in the details of moment-to-moment decision making. (p. 53)

The goal of this study is to begin to uncover the detailed ways that pre-service elementary teachers examine and understand mathematics teaching, the grain size at which they do so, and what constitutes evidence of teaching mathematics for understanding for pre-service mathematics teachers. Thus, the central research questions for this study are: (a) What are pre-service teachers' conceptions of teaching mathematics for understanding? and (b) What counts as evidence for pre-service teachers of their practices in the planning, enactment, and reflection on teaching?

This study takes place in the context of the Performance Assessment for California Teachers (PACT), also known as the Teaching Event. The PACT is a standards-based performance assessment designed to measure pre-service teacher learning (Pecheone, Pigg, Chung, & Souviney, 2005). The assessment draws on a variety of data sources, such as lesson plans, student work, videos of teaching, and teacher reflections to measure pre-service teachers' ability to plan, implement, assess, and reflect on an integrated series of lessons that are intended to facilitate quality instruction for all learners. Furthermore, this assessment is broken down by content area, with each defining the particular knowledge and practices that are valued by the discipline. An important feature of the assessment for all disciplines is its focus on application of subject-specific pedagogical knowledge. For mathematics in particular, candidates are prompted to design, enact, and reflect on a series of lessons directed toward building conceptual understanding, computational and procedural fluency, and mathematical reasoning skills (Pecheone & Chung, 2006). Because the vision of mathematics teaching and learning promoted by the PACT is consistent with that advocated by research on teaching mathematics for understanding, we use this set of artifacts to study the specific ways that prospective teachers have come to understand this approach to mathematics instruction.

To begin to investigate these questions, we conducted a case study of four pre-service teachers' PACT materials, including their written documents and video of teaching. These case studies provide insight into how pre-service teachers make sense of mathematics instruction and how they attempt to implement teaching practices to promote understanding of mathematics. We propose that the results of this study can be used to construct a framework to examine pre-service teachers' understanding of mathematics instruction more broadly, as well as to inform the design of teacher education programs that promote learning to teach mathematics for understanding.

Learning and Teaching Mathematics for Understanding

We frame this study by first reviewing the construct of learning mathematics for understanding and then consider the implications for pre-service teacher learning. Drawing on mathematics education research (Carpenter, Fennema, Franke, Levi, & Empson, 1999; Fennema & Romberg, 1999; Hiebert et al., 1996, 1997; NRC, 2001), we conceive of learning mathematics for understanding as knowledge of and proficiency with mathematical concepts and procedures, as well as an ability to reason about and make sense of mathematics. In particular, conceptual understanding refers to comprehension of mathematical concepts, operations, and relations; procedural fluency includes skill in carrying out procedures flexibly, accurately, efficiently, and appropriately; and adaptive reasoning involves the capacity for logical thought, reflection, explanation and justification (NRC, 2001, p. 5). However, the typical model for teaching mathematics in American classrooms is to focus on developing knowledge and proficiency with mathematical procedures, with much less focus on making sense of and reasoning about the mathematics (Stigler & Hiebert, 1999). Thus, to make progress on helping both pre-service and practicing teachers adopt a more conceptually focused approach to mathematics teaching, research on mathematics instruction has identified several features of classrooms that achieve this vision of mathematics teaching and learning (Carpenter & Lehrer, 1999; Fennema & Romberg, 1999; Hiebert et al, 1997; National Council of Teachers of Mathematics [NCTM], 2000; Stein, Smith, Henningsen, & Silver, 2000). These include the nature of learning tasks, the role of the teacher, the social culture of the classroom, the type and use of mathematical tools, and the accessibility of mathematics to every student (Hiebert, et al., 1997). Together, these elements can provide a classroom context that promotes mathematical proficiency beyond procedural facility.

To begin, the nature of the tasks that students encounter defines the mathematics they learn. What task mathematics students are asked to work on and how they are asked to work on it shapes how they come to understand the discipline. Furthermore, the kinds of tasks that teachers present, the problems they pose, as well as how they are enacted in teaching, provide different learning experiences for students. As Stein, Smith, Henningsen, and Silver (2000) describe, different tasks require different levels and kinds of thinking, what they refer to as the *cognitive demands* (p. 3). Moreover, they explain that the cognitive demands of a task can change throughout a lesson as it comes to life in the classroom. In classrooms where teaching for conceptual understanding is the goal, tasks should pose genuine problems for students to solve, allow space for students to explore the mathematics and construct solutions based on prior knowledge, and afford opportunities for students to communicate and reflect on their thinking (Hiebert et al, 1997).

A second feature of teaching for understanding involves engaging students in mathematical discourse (National Council of Teachers of Mathematics, 2000). Hufferd-Ackles, Fuson, and Sherin (2004) identify four key components of a math-talk community: questioning, explaining mathematical thinking, source of mathematical ideas, and responsibility for learning. In math-talk classrooms, both the teacher and the students play important roles in posing and answering questions and initiating mathematical ideas that focus on these attributes. Furthermore, students are responsible for articulating and justifying their thinking and for helping each other learn the mathematics. In their analysis of classroom discourse, Schleppenbach, Flevares, Sims, and Perry (2007) examine how student errors are taken up in classrooms where understanding is the goal. They found that errors provide opportunities for inquiry and for more extended conversations to unfold, with teachers and students examining why errors exist, what they reveal about conceptions and misconceptions, and how to go about developing deeper understandings.

As teachers seek to pose tasks and create a discourse environment like we describe, their role shifts in important ways. In traditional classrooms, teachers are typically responsible for presenting mathematical material and being the "knowers" of mathematics, modeling solutions, and checking for correctness. In the classrooms envisioned by reform mathematics, their roles change to posing authentic tasks, managing student talk, and inviting students to participate as sources of mathematical ideas (Huffered-Ackles, Fuson, & Sherin, 2004). Clearly, the teacher plays an important role as the manager of the classroom, but a noteworthy difference is that the teacher establishes classroom norms in which students also play important roles in the learning that takes place.

Thus, in classrooms that promote learning with understanding, the social culture of the classroom is one of a community of learners. Hiebert and colleagues (1997) define four features of a classroom community. First, ideas are the currency of the classroom. That is, they have the potential to contribute to everyone's learning and thus warrant respect. Second, students have autonomy with respect to the methods used to solve problems. There are a variety of methods for solving problems. Students need to be given opportunities to construct solutions, and they should be valued, discussed, and explored together. Third, mistakes are sites for learning. Student errors afford opportunities to examine student reasoning and should be made public for consideration and discussion, rather than corrected by the teacher (Schleppenbach, Flevares, Sims, & Perry, 2007). Finally, correctness and reasonability lie in the logic and structure of mathematics, not in the social structure of the classroom. In other words, the mathematics should drive whether an idea makes sense, not based on the popularity or position of the students who propose the ideas (Hiebert, et al., 1997).

Another core feature of classrooms that promote learning for understanding is that tools are used as learning supports. But tools in and of themselves are not sufficient. As Cohen (1990) illustrates, tools can be incorporated into a mathematics lesson, but teachers can adopt them in ways to continue to promote traditional teaching practices. In the model of teaching mathematics for understanding, tools are viewed as supports for learning. They are useful in helping students solve problems, represent their thinking, and communicate their ideas. Further, the kinds of tools students use shape their learning and understanding of the mathematics. For example, whether students use base-ten blocks, hundreds charts, or counting by tens to understand tens and ones groupings will influence what they come to understand and the strategies they use to think about the mathematics. For future teachers, this means developing their understanding of what different tools afford and how they can be used to support thinking and learning.

Finally, in classrooms that promote understanding, issues of equity and accessibility come to the fore. Researchers in mathematics education reform advocate for all students to have access to opportunities to learn mathematics for understanding. That is, tasks need to be established that are appropriate for students of all levels and they should all be expected to engage with the mathematics in the same ways, constructing solutions, communicating their thinking, and reflecting on their mathematical understanding.

Pre-Service Teacher Learning: Learning to Teach Mathematics for Understanding

While teacher education programs strive to help future teachers understand these core dimensions of mathematics teaching and learning, research points to several challenges that prospective teachers encounter in learning to teach mathematics for understanding. First, prior research shows the critical role that pre-service teachers' beliefs and knowledge play in learning to teach (Borko et al., 1992; Eisenhart et al., 1993; Richardson, 1996). Richardson (2003) explains that pre-service teacher candidates bring strong beliefs to their teacher education experience about teaching and learning, and these beliefs play a central role for making sense of what they study. In particular, prospective teachers enter teacher education programs with "highly idealistic, loosely formulated, deeply seated, and traditional" beliefs (Richardson, 2003, p. 6). Thus, an important goal of teacher education is providing prospective teachers with experiences that call their beliefs into question in order to develop alternative images of the learning possibilities for students.

Furthermore, teacher content knowledge also influences practice. For example, Borko and colleagues (1992) describe the case of a student teacher, Ms. Daniels, learning to teach mathematics with the goal of developing students' conceptual understanding. This study examined Ms. Daniels' knowledge in particular and found that while her beliefs were in line with those of the teacher education program, she did not have a sufficient understanding of the content to teach for conceptual understanding. More recent characterizations of teachers' content knowledge for teaching (Ball, Thames, & Phelps, 2008) reveal the depth and range of knowledge required to plan and enact instruction that promotes mathematical understanding and sense-making.

Teacher reflection also plays a central role in learning to teach for understanding. The NCTM *Principles and Standards for School Mathematics* (2000) advocate that teachers have opportunities to continually reflect on and improve their practice. To do so, they should be able to analyze their teaching and consider how their teaching affects student learning. However, this is not a simple matter for pre-service teachers. They need to learn how to deconstruct and analyze teaching as it relates to their learning goals (Hiebert, Morris, Berk, & Jansen, 2007). Recent research on teacher *noticing* (van Es & Sherin, 2002; Jacobs, Lamb, Philipp, Schappelle, & Burke, 2007; Sherin, 2007; Star & Strickland, 2008), similar to what Schön (1983) terms *reflection-in-action*, refers to teachers' in-the-moment decision making and suggests that both what teachers attend to in teaching and how they reason about it influences the learning opportunities that arise for students. However, pre-service teachers have few skills at observing teaching; thus, they need to learn to identify and unpack the interactions in teaching that both support and hinder learning for understanding.

Recent assessments of pre-service teacher learning, such as the PACT Teaching Event, afford the opportunity to examine pre-service teachers' conceptions of teaching mathematics for understanding. First of all, they document pre-service teachers' knowledge and practice in the context of the work of teaching. Second, they prompt teachers to examine dimensions of classrooms that promote teaching for understanding. Third, pre-service teachers use artifacts from practice as evidence for claims they make about their instruction as it relates to the goals of the assessment. Thus, using the Teaching Event, we can investigate what teachers have come to understand about mathematics teaching and learning through their planning, enactment, and reflection on teaching.

Research Study

Study Context: Pre-Service Teacher Assessment

Teacher assessment has been in place for centuries, and as Shulman (1986) pointed out, early assessments privileged content knowledge for teaching and later shifted to focus primarily on the pedagogical knowledge and skills for teaching (Shulman, 1986). More recent assessments, however, acknowledge the important role of content and general pedagogical knowledge, as well as specialized pedagogical-content knowledge needed for teaching, and they have been designed to integrate and evaluate teachers' knowledge and skills in these areas. Sample assessments include the National Board for Professional Teaching Standards for practicing teachers, and for pre-service teachers, the Connecticut Teacher Assessment Center Program (CONNTAC) and California's Performance Assessment for California Teachers (PACT) Teaching Event. These assessments are in the form of portfolios, and they prompt teachers to attend to the development of student learning over the course of a learning sequence, with attention to the relationship between the particular teaching moves they employ and the learning that results. This analysis requires knowledge of students and their learning; the subject matter and how to teach particular content; how to manage and monitor student learning of diverse student populations; and skills at reflecting and inquiring into one's practice.

The PACT Teaching Event draws from artifacts created while teaching, along with accompanying commentary that provides the context for examining and interpreting the artifacts. For the mathematics Teaching Event, candidates are prompted to plan instruction and assessment of 3-5 lessons to develop students' conceptual understanding, computational/procedural fluency, and mathematical reasoning skills. This assessment places student learning at the center, with particular attention to subject-specific pedagogy and the teaching of English Learners. Candidates submit a portfolio with both written and video components, documenting a brief learning segment that consists of several parts: Context, Planning, Instruction, Assessment, Reflection, and Academic Language. Subject-specific pedagogy is emphasized through the analytic prompts for the learning segment and video clips. Throughout the Teaching Event, candidates are prompted to articulate, explain, and justify planning and teaching decisions as they relate to teaching mathematics for understanding. In particular, they are asked to explain why the content is important for learning the mathematics, how they planned and taught a lesson to promote students' development of conceptual understanding, computational and procedural fluency, and mathematical reasoning skills, as well as their participation in mathematical discourse. Furthermore, they are prompted to discuss both the development of student learning over the course of the lesson sequence and how they assessed student learning as it relates to the learning goals. Thus, these foci engage candidates in close examination of their practice as it relates to the development of students' mathematical proficiency. We use this assessment tool then to investigate our research questions because it captures in both written and video form how teachers plan, enact, and reflect on mathematics instruction that has a central focus to teach mathematics for understanding.

Participants and Data Collection

Data for this study came from the 2006-07 cohort of multiple subject teacher education candidates at a large California state university that is a member of the PACT Consortium. In the 2006-07 academic year, the cohort consisted of 80 multiple subject teacher education candidates, and they were required to complete their PACT Teaching Event on a mathematics lesson sequence. For the initial inquiry into how the candidates understood the construct of teaching mathematics for understanding, we selected two of the highest scoring candidate portfolios and the two lowest scoring portfolios (See Table 1). We used this sampling strategy as the initial step to allow us to see the extremes in the ways the candidates planned, taught, and reflected on their mathematics teaching. Furthermore, while the candidates' lesson topics and grade levels differed, the variety of topics was useful because they provided a broad picture of how candidates make sense of the principles for teaching for understanding.

Data for this study consists of three parts of the Teaching Event for these four teachers. These include the lesson plan documents and reflections and commentary on lesson plan design and enactment, the video segments of teaching and reflections on teaching represented in the video segments, and reflections on the overall lesson sequence. The PACT Teaching Event consists of a series of questions to which candidates respond to make a case for effective instruction. In this study, we examined candidates' responses to the following prompts:

- How do key learning tasks in your plans build on each other to support students' development of conceptual understanding, computational/procedural fluency, mathematical reasoning skills, and related academic language? Describe specific strategies that help build student learning across the learning segment. Reference the instructional materials you have included, as needed.

- In the instruction seen in the clip(s), how did you further the students' knowledge and skills and engage them intellectually

Table 1
Participant Information

PACT Assessment	Teacher	Grade Level	Mathematical Focus
Low Scoring	Karen[1]	4th grade	Fractions and mixed numbers; Estimating fractions
	Nick	3rd grade	Geometry—lines, rays, angles, triangle, and perimeter
High Scoring	Melissa	6th grade	Percents, fractions, and decimals
	Lorena	2nd grade	Adding and subtracting three digit number problems; Regrouping

in understanding mathematical concepts and participating in mathematical discourse? Provide examples of both general strategies to address the needs of all of your students and strategies to address specific individual needs.

• Describe the strategies you used to monitor student learning during the learning task shown on the video clip(s). Cite one or two examples of what students said and/or did in the video clip(s) or in assessments related to the lesson that indicated their progress toward accomplishing the lesson's learning objectives.

Responses to these questions from the Teaching Event provide us with insight into the particular ways candidates perceived they designed and implemented instruction that promoted students learning mathematics for understanding.

Data Analysis

Qualitative methods were used to examine how pre-service teachers conceptualize and make claims about teaching mathematics for understanding (Mirriam, 1998; Schoenfeld, Smith, & Arcavi, 1993). In particular, both authors reviewed the four cases and constructed analytic memos (Miles & Huberman, 1994) that characterize the pre-service teachers' thinking and practice along the following dimensions of teaching mathematics for understanding: the nature of mathematical tasks, mathematical discourse, the use of tools for learning, the social culture of the classroom, and norms for participation (Carpenter & Lehrer, 1999; Fennema & Romberg, 1999; Hiebert, et al., 1997; Hufferd-Ackles, Fuson, & Sherin, 2004; Stein, Smith, Hennigsen, & Silver, 2000). These categories were used to inform what areas we focused on in our analysis of the written documents and the videos of teaching. For example, for the area of mathematical discourse, we reviewed the lesson plans and commentary for evidence of the candidate seeking to create an environment where both students and teachers communicate about mathematical ideas (e.g. question, explain, justify, support each other's learning) and examined what in particular they planned to do to create a discourse community and their justification for their plan. We then turned to video clips to study how these plans were put into practice, as a second source of evidence of understanding. While candidates may plan to conduct a class discussion, the enactment of the discussion reveals the roles teachers and students play and how ideas are developed in context. We followed this process for each dimension of teaching for understanding.

Furthermore, research on teacher noticing (Jacobs et al., 2007; van

Es & Sherin, 2002; Star & Strickland, 2008) also informed the analysis. In particular, this research provides a framework for characterizing what teachers examine as they observe teaching and how they reason about teaching and learning, both in the act of teaching and when they reflect on teaching. What candidates attend to refers to the issues they notice (classroom management, mathematical thinking, classroom discourse, pedagogy, and classroom climate) as well as who they notice (the whole class, particular students, or the teacher). How candidates reason about what they observe includes their stance (describe, evaluate, or interpret) and their level of specificity (oversimplify and generalize versus narrow and specific events and interactions). Thus, we used this to guide how we characterized what teachers paid attention to as we examined the video records, as well as their approach to observing teaching as they reflected on the learning segment in the commentary and reflection segments of the Teaching Event. This framework also provided us with a way to capture what counted as evidence for the candidates when they made claims about their instructional practice as it related to the Teaching Event prompts.

For all four teachers, we examined both the written documents (lesson plans and reflections) and video (teaching) and created memos to characterize how they implemented each dimension of classroom environments that support teaching for understanding across the Teaching Event and how they identified these dimensions in their teaching. Once the analytic memos were created, we compared memos for evidence of teaching for understanding and identified several themes that capture differences and similarities in the ways that high and low performing candidates interpret this construct. To be clear, we do not expect that these are the only ways that pre-service candidates will define the construct of teaching for understanding. Rather, the themes we identified emerged from the cases herein. We argue that this analysis is useful because it begins to articulate how prospective teachers understand the practices for teaching and learning mathematics for understanding, specifically, what features of a classroom environment should be present as they plan for instruction, as well as how they enact these features in teaching. Furthermore, these results can be used to inform analysis of additional cases in order to create a more robust characterization of pre-service teachers' understanding of this area of focus of the Teaching Event. We now present the results from this analysis.

Results & Discussion

Data analysis reveals several differences between the low and high performing candidates' conceptions of teaching and learning mathematics for understanding. These differences fall within three categories: the use of mathematical tools; the roles of students and teachers in the learning setting; and pre-service candidates' analyses and reflections on teaching.

Different Conceptions of Teaching Mathematics for Understanding: Use of Mathematical Tools

A central element of classrooms that promote learning mathematics for understanding is the use of tools. In mathematics classrooms, tools, such as base-10 blocks or Unifix cubes, are used to represent mathematical ideas and to help students make connections between procedures and the underlying mathematics. In the Teaching Event portfolios we analyzed, Melissa and Lorena used tools to create multiple representations of the mathematics, whereas in the cases of Nick and Karen, the tools were selected because they were familiar to students or they believed the students would enjoy them. For example, in Lorena's classroom, her students worked on multi-digit addition and subtraction. She elected to use base-10 blocks because they represent hundreds, tens, and ones and students can manipulate them in order to regroup them and build their understanding of place value.

In contrast, in Nick's classroom, he taught a lesson on different kinds of triangles, and in the discussion of acute and obtuse triangles, he explains that he will alter his voice in different ways to represent these triangles.[2] In his lesson plans, he writes,

> A good way to remember that an obtuse angle is greater than a right angle is to practice saying it in a huge way. Model to students how to say obtuse in a low and big voice… Next, tell the students that a better word for less than angle is acute angle. Ask the students how we might say acute angle. Model for the students how to say acute in a high mouse like voice… (p. 16)

This verbal representation, in this case changing one's voice to represent different size triangles, has little, if anything, to do with the mathematics. To justify this design decision, Nick explains in his reflection of the lesson sequence that his lesson sequence design is informed by Gardner's notion of multiple intelligences. Thus, he uses the two different voices to appeal to auditory learners. Furthermore, he explains that he makes choices because students will think it is fun. We can interpret that this choice was made because of the emotional impact it would have on students. This example is problematic from the perspective of developing conceptual understanding because it does not help students make sense of or reason about the different properties of triangles in order to distinguish them in the future. Thus, this becomes a superficial representation and lacks mathematical meaning to promote student understanding. Choosing tools that are mathematically relevant versus ones that are not was a consistent distinction we observed between the low and high performing candidates. While all four candidates have some understanding that tools are an important dimension of the learning environment, they differ in what they understand the tools should be used for and how they promote learning the mathematics. The higher scoring cases think of tools as supports for learning or as a means for representing mathematical reasoning or solutions. The lower scoring candidates consider tools in light of their entertainment value and make unwarranted connections between the use of tools and learning for understanding.

Lorena, for example, uses base-ten manipulatives alongside the traditional representation of an addition or subtraction problem. She adds coins because of the base-ten nature of United States money system as another way for students to understand base-ten operations. It is noteworthy that in Lorena's reflections she admits that her cooperating teacher restricted her use of tools and that she would, "allow students to have some time exploring and working with the manipulatives. . . . [and] to experiment with adding and subtracting numbers using invented strategies that they developed." At the beginning of Nick's lesson on triangles, he introduces students to lines, rays, and angles and asks students to position their arms in different ways to represent each. When he reflects on this strategy, Nick states that he would do more of the same with the use of students' arms, and he writes, "make a little dance or sequence of motions with the arm movements. Students could chant the names of the lines, rays, or angles as the students move through the motions. This will also help to further develop academic language and increase student discourse." While Lorena is restricted by the context of her teaching, she shows an understanding of the potential value of mathematical tools, while Nick's understanding of tools as mathematical supports is limited.

The Roles of Teachers and Students in the Learning Environment

A second difference between the low and high performing candidates' portfolios relates to the roles individuals adopt in promoting mathematical thinking. A key dimension to engaging students in developing their own understanding of mathematics involves shifting the responsibility for learning solely from the teacher to both the students and the teacher (Carpenter & Lehrer, 1999; Hufferd-Ackles, et al., 2004). In the analysis of the written documents and the videos, we observed a difference between how the teacher candidates intended to position themselves to the learners and how they actually did so in their video. Throughout the planning documents for all four candidates, they appeared to be planning for students to take on responsibility for their learning. However, an important difference between the high and low scoring candidates was that the higher-scoring candidates noticed discrepancies in their planning and teaching on this dimension in their reflection, while the lower-scoring candidates did not. For example, one of the high-scoring candidates, Melissa, writes in her reflections that in the future she will "invite the students to be more involved in the lesson. I guided them a little too much through the lesson and I think they would have benefited more by figuring out the process to arrive to the solution by themselves instead of me explaining every step." This is in contrast to Nick, who resolves to add more of the same, instead of noticing that his students were virtually silent even though he had planned for students to "take responsibility for what they say, as they are sharing information with another student."

Even though they might not have been fully effective or cognitively demanding, efforts to involve students in their own learning were visible in the high-scoring cases. For example, we observed attempts on both Melissa's and Lorena's parts to shift the ownership of the lesson to the students, by inviting students to assist with representing a problem at the board or probing student ideas. In contrast, both Nick and Karen controlled the lessons and did much of the mathematical work that we observed in the video segments. Furthermore, in the reflections on teaching, both Melissa and Lorena showed an awareness of what students were doing and saying throughout the lessons, whereas Nick and Karen focused consistently on themselves and on their teaching. For example, in Lorena's lesson commentary, she reflects that if she were to re-teach her lesson, she would make several changes to allow for more student interaction, such as giving more time to work with manipulatives, providing students with opportunities to invent strategies for solving addition and subtraction problems, and offering students a chance to look for patterns and make observations about math instead of having them do multiple problems. Similarly, Melissa acknowledges the need to offer students greater freedom to converse with one another about the mathematics, let student questions guide her instruction, and broaden the participation of students from one to many by using participation strategies that increase accountability. This attention to how students interact with and engage with the mathematics was not evident in the low performing candidates' reflections.

This distinction is important because it reveals one perception that the construct of teaching and learning for understanding is relational, that it involves examining both the teacher and the learners and how they interact with each other and with the mathematics. In contrast, Nick's and Karen's portfolios reveal a focus on the teacher as the central agent and students as recipients, with teacher moves used as evidence of engaging students with mathematical procedures and concepts.

Pre-service Teachers' Approaches to Analysis and Reflection

Finally, we observed a difference in how the low and high performing candidates reflected on their teaching mathematics for understanding. Specifically, Melissa and Lorena used specific evidence to support their claims about particular practices in action in the video clip, whereas Nick and Karen referred to little if any of the particular events in the clips to support their claims. Consequently, Melissa and Lorena achieved more substantive analyses, grounded in the particulars of their video clips. In contrast, Nick and Karen over-generalized, offering superficial, global claims without pinpointing specific events or interactions to support claims of best practices in their teaching. We highlight these distinctions because research shows that attention to the specific events that occur in teaching helps promote a more student-centered approach to instruction (van Es & Sherin, 2005).

We propose that these habits of reflection influence the extent to which future teachers notice particular aspects of the learning environment and how they reason about what they see. For example, adopting a finer grained focus on student thinking enables future teachers to notice particular ways that students work with and use manipulatives and draw inferences about how their particular strategies support or constrain learning. Similarly, by attending to the particular ideas students raise in a discussion, they can draw claims about student conceptual understanding. In contrast, a more broad, global focus would facilitate noticing that the students *use* manipulatives but may inhibit them from seeing *how* they use them to engage with mathematical ideas. Likewise, from the broader perspective, candidates may notice students participating in a discussion, but without delving into the particulars of what they say or how they participate, they may draw weaker inferences about what constitutes mathematical discourse and how it advances student learning.

These contrasting perspectives of what counts as evidence of teaching and learning for understanding demonstrated by our four cases suggests that attention should be given in pre-service teacher education to defining core concepts in such a way that their key characteristics are vividly understood through multiple iterations and discussions by candidates. In other words, helping candidates "see" students developing conceptual understandings and reasoning about mathematics must be scaffolded in the same way as developing the candidates' understanding the mathematics itself.

Similar Conceptions of Teaching Mathematics for Understanding

While we identified three ways the pre-service teachers differ in what and how they observed teaching mathematics for understanding, we also found two similarities in their Teaching Events. The first similarity concerns the language the candidates use to discuss teaching and learning for understanding, and the second involves how the four teachers planned, enacted, and reflected on their efforts to promote mathematical explanation and reasoning.

Appropriating a Discourse of Teaching and Learning

One similarity between the four candidates' PACT assessments was that they attempted to use the language they had been introduced to in the teacher credential program to write about and reflect on mathematics teaching and learning. In particular, when discussing how they designed instruction to promote mathematical understanding, they referred to the importance of group work (e.g., Think-Pair-Share or Think-Talk-Show), the use of tools and manipulatives, accessing students' prior knowledge, making real world connections, and differentiating instruction. These phrases and concepts were evident across the planning and reflection documents and suggest that the prospective teachers had come to appropriate a discourse they had learned in the teacher education program to talk about their teaching and student learning. However, as we discussed above, the distinctions in their understandings of these constructs became apparent, for example, in how they put these constructs into practice, as well as in the depth of their analysis and reflection of their teaching.

Mathematical Discourse and Reasoning

The second similarity concerns the dimension of pressing students to explain and reason through the mathematics. Research highlights the importance of students explaining and discussing their thinking and teachers questioning students, all in the service of helping students reason and make sense of the mathematics (Carpenter & Lehrer, 1999; Hufferd-Ackles, et al., 2004). Our analysis found that the four candidates held similar notions of engaging students in reasoning about and explaining mathematical ideas. Consistent across all four cases was that the candidates posed questions in which the answer was one of three types: a) yes or no answers, such as "Does anyone have any questions?" or "Does everyone get this?"; b) a numerical answer to the math problem being posed; or c) the next step in a mathematical procedure being led by the teacher. While all four teachers asked questions, their questions were rarely of the sort to promote deep sense-making and problematizing of the mathematics (Hiebert, et al., 1996).

Furthermore, they did not create a discourse community in which student talk was focused on reasoning. In fact, all four candidates appear to have unclear and vague notions of what it means to engage students in mathematical discourse that promotes reasoning and thus use weak evidence to make claims that it was evident in their teaching. For instance, in Nick's instructional commentary, he writes, "students participated in discourse by silently communicating with arm positions. Discourse was also achieved through verbal and written explanations of mathematical concepts. All of the angles, rays, and lines discussed in class were drawn on the board and labeled accordingly." His evidence of mathematical discourse includes students communicating through gestures as well as visual images represented and labeled on the board for the students to see. While these are both ways of communicating information, in neither case were students placed in positions to explain the gestures they made and why they made sense for the mathematics they were learning or to compare and contrast the visual images on the board and the distinctions between each. Thus, students were presented with images, but the images were not used to engage students in thinking about the mathematical concepts they represent.

The two high scoring candidates also had vague and superficial conceptions of developing a mathematics discourse community. For example, in Lorena's teaching, she sets up structures for students to work together to solve the subtraction problems that involve regrouping, but there is not evidence of Lorena engaging the students in talk about their work. Furthermore, her reflections do not point to the need for more student discourse. On the other hand, Melissa acknowledges the value of students talking to one another. She writes in her reflection, "I think the students would have benefited by performing this activity in a Think, Pair, Share. This way they would help one another solve the problem and could answer each other's questions." While she seems to believe that student conversation would be useful, her focus is less on the discourse about the mathematics and more on the participant structure that could be used to organize student talk. Thus, we see that both high and low candidates have fragile understandings of discourse community features.

Conclusion

In sum, this study sought to understand how pre-service teachers have come to understand the construct of teaching mathematics for understanding, with particular concern for how they define conceptual understanding and reasoning mathematically, as well as what they count as evidence of this practice in teaching. We found that the four cases we analyzed point to distinctions and similarities in how candidates conceptualize this construct, and these variations are useful for beginning to define a framework for pre-service teachers' conceptions of teaching mathematics for understanding. In particular, we identified three differences in relation to the use of mathematical tools, the roles of teachers and students in the learning environment, and strategies for analyzing and reflecting on teaching and learning. We also identified two areas of similarity, namely, the language candidates use to talk about mathematics teaching and learning and the ways they consider engaging students in mathematical discourse and reasoning. These results are important because they highlight the particular ways that pre-service teachers have come to make sense of critical dimensions of mathematics instruction, as well as how difficult it is to implement in practice, even if they can talk about it in their planning and reflection.

This study begins to uncover the ways that pre-service teachers examine teaching, the grain size at which they do so, and how they use their analysis and interpretations of their practice to argue for evidence of effectively engaging students with mathematical concepts and in mathematical discourse. However, we observed important distinctions both in how the high and low performing candidates have come to define teaching mathematics for understanding and in what they count as evidence for this in practice. Understanding these distinctions can help teacher educators design instruction that scaffolds pre-service teachers to develop richer and more nuanced understandings of teaching mathematics for understanding and to learn to design instruction to accomplish this goal in their teaching practice.

Notes

[1] Pseudonyms are used to protect the identity of teacher candidates.

[2] Drawing on Cole (1996) and Vygotsky (1978), we define tools as physical objects and visual representations, as well as language and its use in context.

References

Ball, D., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching. *Journal of Teacher Education, 59*(5), 389-407.

Boaler, J., & Humphreys, C. (2005). *Connecting mathematical ideas: Middle school video cases to support teaching and learning*. Portsmouth, NH: Heinemann.

Borko, H., Eisenhart, M., Brown, C. A., Underhill, R. G., Jones, D., & Agard, P. C. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? *Journal of Research in Mathematics Education, 23*, 194-222.

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.

Carpenter, T. P. & Lehrer, R. (1999). Teaching and learning mathematics with understanding. In E. Fennema & T. A. Romberg (Eds.), *Classrooms that promote mathematical understanding* (pp. 19-32). Mahwah, NJ: Lawrence Erlbaum Associates.

Cohen, D. K. (1990). A revolution in one classroom: The case of Mrs. Oublier. *Educational Evaluation and Policy Analysis, 12*(3), 311-329.

Cole, M. (1996). *Cultural psychology: A once and future discipline*. Cambridge, MA: Harvard University Press.

Eisenhart, M., Borko, H., Underhill, R., Brown, C., Jones, D. & Agard, P. (1993). Conceptual knowledge falls through the cracks: Complexities of learning to teach mathematics for understanding. *Journal for Research in Mathematics Education, 24*(1), 8-40.

Fennema, E. & Romberg, T. A. (Eds.). (1999). *Classrooms that promote mathematical understanding*. Mahwah, NJ: Lawrence Erlbaum Associates.

Hatfield, M. M., & Bitter, G. (1995). *Understanding teaching: Implementing the NCTM professional standards for teaching mathematics.* (CD-ROM). Technology based learning and research, Arizona State University, Tempe, AZ.

Hiebert, J., Carpenter, T., Fennema, E., Fuson, K., Human, P., Murray, H., Oliver, A., & Wearne, D. (1996). Problem-solving as the basis for reform in curriculum and instruction: The case of mathematics. *Educational Researcher*, *25*(4), 12–21.

Hiebert, J. Carpenter, T., Fennema, E., Fuson, K. C., Wearne, D., Murray, H., Olivier, A., Human, P. (1997). *Making sense: Teaching and learning mathematics with understanding*. Portsmouth, NH: Heinemann.

Hiebert, J., Morris, A. K., Berk, D., & Jansen, A. (2007). Preparing teachers to learn from teaching. *Journal of Teacher Education, 58*(1), 47-61.

Hufferd-Ackles, K., Fuson, K. C., & Sherin, M. G. (2004). Describing levels and components of a Math-Talk Learning Community. *Journal for Research in Mathematics Education, 35*(2), 81-116.

Jacobs, V., Lamb, L. C., Philipp, R., Schappelle, B., & Burke, A. (2007, April). Professional noticing by elementary school teachers of mathematics. Paper presented at the Annual Meeting of the American Educational Research Association Conference, Chicago, IL.

Lampert, M., & Ball, D. (1998). *Teaching, multimedia and mathematics: Investigations of real practice*. New York: Teacher's College Press.

Lortie, D. C. (1975). *Schoolteacher: A sociological study*. Chicago: University of Chicago Press.

Merriam, S. (1998). *Qualitative research and case study applications in education*. San Francisco: Jossey-Bass.

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis* (2nd ed.). Thousand Oaks, CA: Sage.

National Council of Teachers of Mathematics. (2000). *National Council of Teachers of Mathematics principles and standards for school mathematics*. Reston, VA: Author.

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, & B. Findell (Eds.), Washington, DC: National Academy Press.

Pecheone, R. L., & Chung, R. (2006). Evidence in teacher education: the performance assessment for California teachers (PACT). *Journal of Teacher Education, 57*(1), 22-36.

Pecheone, R. L., Pigg, M. J., Chung, R. R., & Souviney, R. J. (2005). Performance assessment and electronic portfolios: Their effect on teacher learning and education. *The Clearing House*, *78*(4), 164-176.

Philipp, R. A. (2008). Motivating prospective elementary school teachers to learn mathematics by focusing upon children's mathematical thinking. *Issues in Teacher Education, 17*(2), 7-26.

Richardson,V. (1996). The role of attitudes and beliefs in learning to teach. In J. Sikula (Ed.), *Handbook of research on teacher education* (2nd ed., pp. 102-119). New York: Macmillan.

Richardson, V. (2003). Preservice teachers' beliefs. In J. Raths & A. McAnench (Eds.). *Advances in teacher education*, *Vol. 6* (pp. 1-22). Greenwich, CT: Information Age Publishers.

Santagata, R., Zannoni, C., & Stigler, J. (2007). The role of lesson analysis in pre-service teacher education: An empirical investigation of teacher learning from a virtual video-based field experience. *Journal of Mathematics Teacher Education, 10*(2), 123-140.

Schleppenbach, M., Flevares, L. M., Sims, L., & Perry, M. (2007). Teacher responses to student mistakes in Chinese and U.S. mathematics classrooms. *Elementary School Journal, 108,* 131-147.

Schleppenbach, M., Perry, M., Miller, K. F., Sims, L., & Fang, G. (2007). The answer is only the beginning: Extended discourse in Chinese and U.S. mathematics classrooms. *Journal of Educational Psychology. 99*, 380-396.

Schoenfeld, A. H., Smith, J. & Arcavi, A. (1993) Learning: The microgenetic analysis of one student's evolving understanding of a complex subject matter domain. In R. Glaser (Ed.), *Advances in instructional psychology, Vol. 4* (pp. 55-175). Majwah, NJ: Lawrence Erlbaum Associates.

Schön, D. A. (1983). *The reflective practitioner: How professionals think in action*. New York: Basic Books.

Sherin, M. G. (2007). The development of teachers' professional vision in video clubs in R. Goldman, R. Pea, B. Barron, & S. Derry (Eds.) *Video research in the learning sciences.* Mahwah, NJ: Lawrence Erlbaum Associates.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. A. (2000). *Implementing standards-based mathematics instruction: A casebook for professional development.* New York: Teachers College Press.

Stigler, J. W. & Hiebert, J. (1998). *The teaching gap: Best ideas form the world's teachers for improving education in the classroom.* New York: Free Press.

van Es, E. A., & Sherin, M. G. (2002). Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. *Journal of Technology and Teacher Education*, *10*(4), 571-596.

van Es, E.A., & Sherin, M.G. (2005, April). The influence of video clubs on teachers' thinking and practice. Paper presented at the Annual Meeting of the American Educational Research Association Conference, Montreal, Canada.

Vygotsky, L. S. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.

Zeichner, K. M., & Liston, D.P. (1987) Teaching student teachers to reflect. *Harvard Educational Review, 57*(1), 23-48